AD-A181 406

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	Approved for public release; distribution unlimited.

DTIC ELECTED JUN 1 1 1987 D

4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
	AFOSR-TR- 87-0790

6a. NAME OF PERFORMING ORGANIZATION	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
Northwestern University	MNE	Air Force Office of Scientific Research

6c. ADDRESS (City, State and ZIP Code)	7b. ADDRESS (City, State and ZIP Code)
619 Clark Street Evanston, IL 60201	Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR	NM	Grant: AFOSR-84-0340

8c. ADDRESS (City, State and ZIP Code) BlDa 410 Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS.			
	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
	61102F	2304	C3	

11. TITLE (Include Security Classification)
MONTE CARLO RELIABILITY ANALYSIS (Unclassified)

12. PERSONAL AUTHOR(S)
Prof. E. E. Lewis

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Annual Technical	FROM 9/1/85 TO 8/31/86	12/1/86	

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Reliability, Monte Carlo, Simulation

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The work carried out during the 1985/86 contract year is summarized: Markov Monte Carlo methods are generalized to include inhomogeneous Markov processes. Two new sampling techniques allow the treatment of reliability problems which include time-dependent failure rates and preventive maintenance. Incorporation of periodic testing and repair allows classes of revealed and unrevealed failures to be combined in problems with wear, periodic maintenance and component dependencies.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Maj. Woodruff	(202) 767-5028	NM

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE.   UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

1985/86 Activity

During the 1985/86 contract period research has been concentrated on the use of inhomogeneous Markov processes to treat time-dependent failure rates and preventive maintenance, and on the effective Monte Carlo simulation of the resulting models. This work is summarized in the enclosed paper which was recently published in Reliability Engineering.

The principal investigator was assisted by Mr. Z. Tu (not supported by AFOSR) in the work on inhomogeneous Markov processes. In addition, the contract supported a Ph.D. student, Mr. F. Boehm. During the contract period Mr. Boehm began to examine two related aspects of Monte Carlo simulation. First, he examined the departures from the Markov condition that are needed to study parts replacement policies. This work is in an active state of development, with computer simulations being carried out presently. It will be reported at the end of the 1986/87 contract year.

In addition, Mr. Boehm is examining more closely the sources of data for wear phenomena that give rise to failure rates that increase with time; his emphasis is on the incorporation of more realistic fatigue failure models into the simulation of mechanical components.

We are interested in applying our simulation methods to problems of active interest to the Air Force. To this end the principal investigator has arranged a trip to Systems Reliability and Engineering Division at the Rome Air Development Center, Griffin AFB, NY. to determine how our methods might interface the ORACLE code system.

87 6 10 228

Monte Carlo Reliability Modeling by Inhomogeneous Markov Processes

E. E. Lewis and Tu Zhuguo*

Department of Mechanical and Nuclear Engineering, Northwestern University,
Evanston, Illinois 60201, USA

ABSTRACT

*Markov Monte Carlo methods for reliability calculations are generalized
to include inhomogeneous Markov processes. Two new sampling tech-
niques allow the treatment of time-dependent failure rates and of
preventive maintenance. Incorporation of periodic testing and repair
models allows classes of revealed and unrevealed failures to be combined in
problems with wear, periodic maintenance and component dependencies.
Numerical illustrations of these phenomena are presented for one, two
and multiple component systems.*

1 INTRODUCTION

In previous papers[1,2] the Lagrangian approach of Markov Monte Carlo
methods has been shown to be very effective for estimating the reliability
and availability of complex systems. The ability to treat general
component dependencies in multicomponent systems, coupled with the
use of variance reduction techniques to greatly increase sampling
efficiency, results in highly efficient algorithms, capable of treating
Markov models that would be intractable by deterministic computational
methods. There are, however, two major limitations on the ability of

*Permanent address: Beijing Nuclear Engineering Institute, PO Box 840, Beijing,
People's Republic of China.

the foregoing Monte Carlo methods in the faithful modeling of reliability problems: they are limited to constant failure rates and to revealed failures. In order to model component aging or wear, and the concomitant effects of preventive maintenance, the formulation must be generalized to include time-dependent failure rates. If failures are unrevealed, then periodic testing also must be included in the modeling.

A perfectly general treatment of wear, maintenance and repair phenomena would require a number of departures from the conditions that define Markov processes. However, a number of the more important of these phenomena can be modeled by generalizing the existing Monte Carlo methods, which are limited to homogeneous Markov processes, to treat inhomogeneous Markov processes. For when the resulting time-dependent transition rates are combined with the ability to make deterministic state transitions at specified time intervals, the resulting simulation can treat wear, preventive maintenance at specified times, and reasonable approximations to the repair of several classes of revealed and unrevealed failures.

In this paper we formulate the required equations and present two methods for transition sampling in the presence of time-dependent transition rates: self-transitions and mode sampling. The procedures are then generalized to allow for periodic testing and repair. Finally, we examine a number of problems for one, two and multicomponent systems in which component dependencies of the load sharing variety are present. For such systems the ability of inhomogeneous Markov Monte Carlo simulation to treat wear, preventive maintenances and combinations of revealed and unrevealed failures is thus demonstrated.

2 THE INHOMOGENEOUS MARKOV EQUATIONS

In order to treat reliability for systems in which component wear and/or early failures are present, we must represent the time dependence of failure rates of real components. These typically are represented in the form of 'bathtub' curves, such as that illustrated in Fig. 1. As in earlier work, however, we also must be able to represent dependencies between component failures, such as occur, for example, with shared loads, shared repair crews and standby configurations. To incorporate time-dependent failure rates into a system with component failure dependencies we generalize our earlier homogeneous Markov formalism into
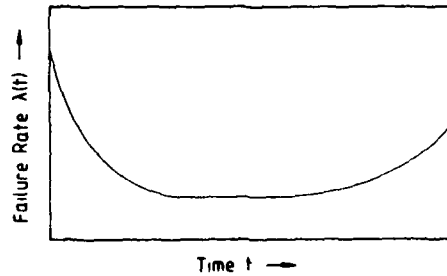
Fig. 1. Failure rate curve exhibiting 'bathtub' behavior.

inhomogeneous Markov formalism: one in which the transition rates between states are explicit functions of time.

To begin, assume that we have a system with N components, each of which may be in an operational or a failed state. There are then 2^N states corresponding to the unique combinations of operational and failed components. We let $p_k(t)$ = probability that the system is in state k at time t, and therefore

$$\sum_k p_k(t) = 1$$

We designate $k = 0$ as the initial state in which all components are operational, and then

$$p_k(0) = \delta_{k0}$$

The equations for the continuous time Markov process governing the probabilities $p_k(t)$ are

$$\frac{d}{dt} p_k(t) = -\sum_{k'} \gamma_{k'k}(t) p_k(t) + \sum_{k'} \gamma_{kk'}(t) p_{k'}(t) \tag{1}$$

where the $\gamma_{k'k}(t)$ are the transition rates between states. More precisely, the system is defined as a semi-Markov process.[3] Each transition leads to a state change. Thus,

$$\gamma_{kk}(t) = 0$$

and the summations do not include the $k' = k$ terms.

It is convenient to rewrite eqn (1) as

$$\frac{d}{dt} p_k(t) = -\gamma_k(t) p_k(t) + \sum_{k'} q(k \mid k', t)\gamma_{k'}(t) p_{k'}(t)$$

where

$$\gamma_k(t) = \sum_{k' \neq k} \gamma_{k'k}(t)$$

and

$$q(k \mid k', t) = \gamma_{kk'}(t)/\gamma_{k'}(t)$$

The quantity $q(k \mid k', t)$ is readily seen to be the conditional probability that given a transition out of state k' at time t, the new state will be k.

In general, $\gamma_k(t)$, the transition rate out of state k, can be represented as

$$\gamma_k(t) = \sum_{l \in O_k} \lambda_{lk}(t) + \sum_{l \in F_k} \mu_{lk}(t)$$

where $\lambda_{lk}(t)$ and $\mu_{lk}(t)$ are the failure and repair rate of component l, and O_k and F_k are the sets of operational and failed components, respectively, in state k. In the case where component dependencies are present, $\lambda_l(t)$ and/or μ_l and/or $\mu_l(t)$ will also depend on the system state k. For example, in a standby configuration the failure rate of the backup unit will depend strongly on whether the primary unit is operational.

The equation for $p_k(t)$ may be put in integral form. By using an integrating factor

$$\exp\left\{ -\int_0^t \gamma_k(t')\,dt' \right\}$$

we obtain

$$p_k(t) = \delta_{k0}\exp\left[-\int_0^t \gamma_0(t')\,dt' \right] + \int_0^t dt' \exp\left[-\int_{t'}^t \gamma_k(t'')\,dt'' \right]$$

$$\times \sum_{k'} q(k \mid k', t')\gamma_{k'}(t')p_{k'}(t')$$

To express this equation in terms of the probability distributions sampled in the Monte Carlo simulations, we introduce

$$\chi_k(t) \equiv \gamma_k(t)p_k(t)$$

which is the probability density of transitions out of state k and multiply by $\gamma_k(t)$ to obtain

$$\chi_k(t) = \int_0^t dt' f(t \mid t', k') \left[\delta_{k0} \delta(t') + \sum_{k'} q(k \mid k', t') \chi_{k'}(t') \right] \qquad (2)$$

where

$$f(t \mid t', k') = \gamma_k(t) \exp\left\{ -\int_{t'}^t \gamma_k(t'') dt'' \right\} \qquad t \geqslant t' \qquad (3)$$

is the probability density that there will be a transition at t given that the system is in state k' at t'.

We may also write this equation in the notation of an earlier paper,[1] by noting that

$$\psi_k(t) = \delta_{k0} \delta(t) + \sum_{k'} q(k \mid k', t) \chi_{k'}(t) \qquad t \geqslant 0 \qquad (4)$$

is the probability density for transitions into state k. The first term on the right is due to the convention that the problem is initialized by a transition into state $k = 0$ at $t = 0$. Combining eqns (2) and (4), we obtain

$$\psi_k(t) = \delta_{k0} \delta(t) + \sum_{k'} q(k \mid k', t') \int_0^t dt' f(t \mid t', k') \psi_{k'}(t')$$

3 MONTE CARLO SAMPLING

If the transition rates are taken to be time-independent, the foregoing equations reduce to the homogeneous Markov formulation. In that case, eqn (3) becomes the exponential distribution and the resulting Monte Carlo sampling for transition times is straightforward. When time-dependent transition rates are present, however, the direct inversion technique used for the exponential distribution[1] is no longer applicable. To illustrate, we first find the cumulative distribution corresponding to eqn (3) to be

$$F(t \mid t', k') = 1 - \exp\left\{ -\int_{t'}^t \gamma_k(t'') dt'' \right\} \qquad (5)$$

To perform direct inversion sampling we set $F(t \mid t', k')$ to a random number ξ uniformly distributed between zero and one. We obtain

$$\int_{t'}^{t} \gamma_k(t'') \, dt'' = -\ln(1 - \xi)$$

The difficulty, of course, is in inverting this expression for t. As an alternative we present two methods for sampling the times between transitions: mode sampling and self-transitions. Following the transition, the sampling for the new state k is straightforward. As in the homogeneous Markov formulation, we simply choose a uniformly distributed random number, say ξ', and then choose the state which satisfied the condition

$$\sum_{k''=0}^{k} q(k'' \mid k', t) < \xi' \leqslant \sum_{k'=0}^{k+1} q(k'' \mid k', t) \tag{6}$$

where

$$q(k' \mid k', t) = 0$$

and t is taken at the time of transition.

Mode sampling

Suppose that the transition rate can be written as the sum of a number of transition modes

$$\gamma_k(t) = \sum_{i} \gamma_k^i(t) \tag{7}$$

Each mode is represented by a two-parameter Weibull distribution with a different exponent. Thus we may write

$$\int_{0}^{t} \gamma_k(t') \, dt' = \sum_{i} (t/\theta_{ki})^{m_i} \tag{8}$$

If we allow several different values of m_i to appear in such series, we can reasonably represent most time-dependent failure rates of interest. For example, the bathtub curve such as that shown in Fig. 1 may be approximated as the superposition of three such terms with $m_1 < 1$,

$m_2 = 1$ and $m_3 > 1$. These correspond to early failures, random failures and aging failures, respectively.

For a multicomponent system all components are assumed to contain the same values of m_i. Then the transition rate given by eqn (7) is written as

$$\gamma_k(t) = \sum_i m_i t^{m_i - 1}/\theta_{k_i}^{m_i}$$

where

$$\frac{1}{\theta_{k_i}^{m_i}} = \sum_{l \in O_k} \frac{1}{\theta_{l_i}^{m_i}} + \delta_{m,1} \sum_{l \in F_k} \mu_l$$

Combining eqns (5) and (8), we may write the cumulative distributions as

$$F(t \mid t', k) = 1 - \prod_i \exp\{-(t/\theta_{k_i})^{m_i} + (t'/\theta_{k_i})^{m_i}\}$$

This may be shown to be a minimum extreme value distribution[4] where the parent distributions are

$$F_i(t \mid t', k) = 1 - \exp\{-(t/\theta_{k_i})^{m_i} + (t'/\theta_{k_i})^{m_i}\} \tag{9}$$

Thus we sample each of the F_i for t_i, the time of the next transition by mode i, and take the minimum value. Hence, for mode i we choose a uniformly distributed random number ξ_i, and set it equal to F_i. The inversion of eqn (9) leads to

$$t_i = \theta_{k_i}\{-\ln(1 - \xi_i) + (t'/\theta_{ki})^{m_i}\}^{1/m_i}$$

The transition is then taken at

$$t = \min_i t_i$$

Self-transitions

In this method we subtract and add the term $\gamma_{kk}(t)p_k(t)$ to the right of eqn (1). This has no effect on the solution. However, the term $k' = k$ is now included in the sums appearing in eqn (1) and in all succeeding equations. We refer to these as self-transitions because they represent transitions back into the same state from which the transition originated.

In effect we have transformed the equations from a semi-Markov to a Markov formulation

Now suppose we choose $\gamma_{kk}(t)$ such that

$$\gamma_{kk}(t) = \gamma_k^0 - \sum_{k' \neq k} \gamma_{kk'}(t)$$

where γ_k^0 is a non-negative constant. We thus have

$$\gamma_k(t) = \gamma_k^0$$

and the modified equations become

$$\frac{d}{dt} p_k(t) = -\gamma_k^0 p_k(t) + \sum_{k'} q(k \mid k', t)\gamma_{k'}^0 p_{k'}(t)$$

This transformation enables us to write the succeeding equation in terms of the exponential probability density

$$f(t \mid t', k) = \gamma_k^0 e^{-\gamma_k^0(t-t')}$$

which may be sampled using a single random number ξ:

$$\xi = F(t \mid t', k') = 1 - e^{-\gamma_k^0(t-t')} \qquad t > t'$$

to obtain the time of the next transition as

$$t = t' - \frac{1}{\gamma_k^0} \ln(1 - \xi)$$

To determine the new state of the system, we again use eqn (6). Now, however, with

$$q(k \mid k'', t) = \gamma_{kk''}(t)/\gamma_k^0$$

The diagonal term $q(k' \mid k', t)$ now is greater than zero. If the transition $k' \rightarrow k'$ is sampled, then the system remains in the same state at t and the calculation continues. Otherwise, a new state k is obtained.

4 COMPONENT MODELING

With either of the sampling methods discussed in the preceding section we may treat wear, preventive maintenance and repair, provided we are able to model them within the framework of inhomogeneous Markov

processes. We first consider wear and then preventive maintenance for situations where there is no repair. We then consider repair, first of revealed and then of unrevealed failures. A revealed failure is one that is known immediately; the modeling of repair is through an exponential distribution of times to repair. An unrevealed failure is one that remains in effect until the system is tested for failure and repaired at some predetermined time intervals. We illustrate each of these models for a one-component system. In most cases, the generalization to multi-component systems, and to systems with component dependencies, is treated analogous to that discussed previously[1,2] for homogeneous Markov processes.

Wear and preventive maintenance

In the case of wear without preventive maintenance, the failure rate curve is likely to appear as in the solid line on Fig. 2. To estimate the reliability of the same component in the case where preventive maintenance is performed at intervals T, we assume that the component is restored to an as-good-as-new condition. The failure rate is thus given by

$$\lambda^*(t) = \lambda(t - NT) \qquad NT \leqslant t \leqslant (N + 1)T$$

The failure rate thus has a periodicity as indicated by the dashed line in Fig. 2.

To treat the periodicity of preventive maintenance, mode sampling must be applied one interval at a time in the following manner. The distribution is sampled to determine whether a failure takes place in the interval $0 \leqslant t \leqslant T$. If it does not, then time is set equal to T, and preventive maintenance is assumed to take place. Then mode sampling
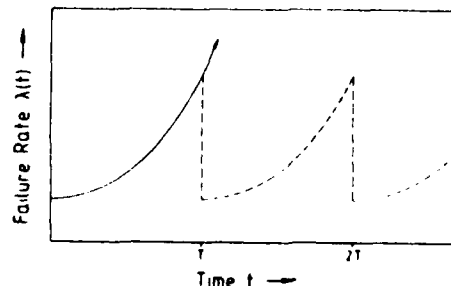

Fig. 2. Failure rate curve illustrating wear and periodic maintenance

is used to determine if a failure takes place in the interval $T < t \leqslant 2T$ and so on. We hereafter refer to mode sampling as method a.

The self-transition method may be performed in a similar manner. Sampling is first carried out to determine whether failure takes place in the interval $0 \leqslant t \leqslant T$. If it does not, time is set equal to T and the interval $T \leqslant t \leqslant 2T$ is considered. We refer to this as method b. Alternately, we may simply apply self-transition to the entire problem domain. The exponential is sampled to determine the next transition regardless of the interval into which it falls. This is referred to as method c.

Revealed failures

If the component fails at time t_f, and the failure is revealed immediately, then a distribution of repair times will be sampled to determine the time of repair t_r. The question must then be decided as to what component failure rate should be used following repair. There are three obvious models, all of which reduce to the standard revealed failure model for the time-independent failure rates of homogeneous Markov processes.

If the repair of a revealed failure to an as-good-as-new state, then, following the failure, the failure rate is set back to the time zero value; this model is indicated by line a in Fig. 3. Secondly, if the repair is made to an as-good-as-old state, then the failure rate is set back to the time at which the failure took place. This model, indicated by line b, assumes that no additional wear occurred during the downtime for repair. Finally, in the continuous wear model we assume that the original failure rate curve, indicated by line c, is followed, thus assuming that the wear process continues unabated through the repair interval.

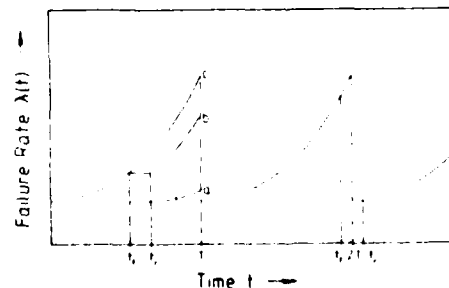Of the three models for revealed failures, only that for continuous

Fig. 3. Failure rate curves showing three models for repair of revealed failures: (a) as-good-as-new; (b) as-good-as-old; (c) continuous wear.

wear falls within the framework of inhomogeneous Markov processes, for only it has a component failure rate that is independent of repair history. However, since $t_r - t_f$, the time interval during which the system is in a failed state, is normally short, the difference between curves b and c should be small; it amounts only to taking no credit for the fact that the aging is slightly reduced due to the downtime for repair. In the calculations that follow we model all revealed failure as continuous wear, it being a good but pessimistic approximation to the as-good-as-old case. Failures may occur such that scheduled maintenance takes place before repair is completed, as indicated by t'_f and t'_r in Fig. 3. In such cases the solid curve is used following repair. The current model neglects downtime for preventive maintenance.

Unrevealed failures

For unrevealed failures the component remains in a failed state until repair or maintenance takes place at some predetermined time interval T. For these failures the repair/maintenance may also be represented as-good-as-new, as-good-as-old or continuous aging models. These are represented respectively by curves a, b and c in Fig. 4, where it is again assumed that the failure occurs at t_f. In this case the continuous aging model represents a poor approximation to as-good-as-old repair. With preventive maintenance, however, restoration to an as-good-as-new state often offers a reasonable model.

For unrevealed failures the as-good-as-new model, a, falls within the inhomogeneous Markov criterion, provided we assume that preventive maintenance is performed at each test interval to return the component to an as-good-as-new condition. The as-good-as-old criterion, b, implies
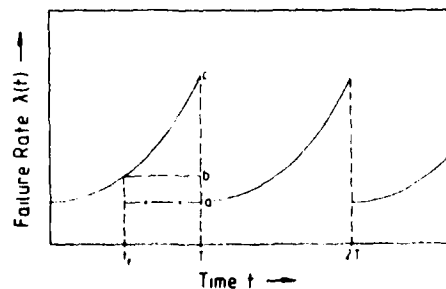

Fig. 4. Failure rate curves showing three models for repair of unrevealed failures: (a) as-good-as-new; (b) as-good-as-old; (c) continuous wear.

that the testing and repair do not effect wear mechanisms. Since it violates the Markov criteria, it is not employed in the calculations that follow. While the continuous aging model falls within the inhomogeneous Markov framework, it is not considered further for unrevealed failures, for with it one must presume that component age accumulates at the same rate regardless of whether it is in a failed state, even for a long period of time. Hence in what follows all tests for unrevealed failures are assumed to include the maintenance required to return the component to an as-good-as-new state.

5 NUMERICAL RESULTS

In this section we first examine the sampling methods developed above, along with the methods for treating revealed and unrevealed failures, in a one-component system. The effects of wear, preventive maintenance and repair models are then applied to a two-component system. Finally, a ten-component system is used to demonstrate the application of the Monte Carlo models to more realistic configurations in which wear, component dependencies and combinations of revealed and unrevealed failures are present. In the calculations that follow, importance sampling in the form of both forced transitions and failure biasing[1] is employed to reduce variance and improve computational efficiency. The application of these variance reduction techniques as well as the procedures for making reliability and availability estimates are identical to those used in homogeneous Markov Monte Carlo.[1,2] Unless otherwise specified, all results are based on runs of 10 000 histories. In no case is more than a few seconds required on a Control Data Cyber 205 in scalar mode.

Single component

The sampling methods for the time to transition are applied to a component with a failure rate given by

$$\lambda(t) = \lambda_0 + (m/\theta)(t/\theta)^{m-1} \qquad (\mathrm{yr}^{-1}) \tag{10}$$

where we use $\lambda_0 = 0.013/\mathrm{yr}^{-1}$, $\theta = 7.5\,\mathrm{yr}$ and $m = 2.5$. In Table 1 the unreliability $\tilde{R} = 1 - R$ is given for a 5-yr design life, where N is the number of intervals into which the design life is divided for purposes of performing as-good-as-new preventive maintenance. Thus for $N = 5$ preventive maintenance is performed annually. The reference results are

TABLE 1

Single-component Unreliability as a Function of Preventive Maintenance Strategy

	N	1	2	3	4	5
$\bar{R}$	Ref.	0·3481	0·1757	0·1261	0·1045	0·0928
	Method a	0·3522	0·1762	0·1309	0·1060	0·0919
	Method b	0·3496	0·1792	0·1291	0·1067	0·0946
	Method c	0·3496	0·1780	0·1266	0·1056	0·0935
Time(s)	Method a	$0·1277 \times 10^1$	$0·1732 \times 10^1$	$0·2274 \times 10^1$	$0·2834 \times 10^1$	$0·3403 \times 10^1$
	Method b	$0·1342 \times 10^1$	$0·1275 \times 10^1$	$0·1225 \times 10^1$	$0·1188 \times 10^1$	$0·1158 \times 10^1$
	Method c	$0·1297 \times 10^1$	$0·1204 \times 10^1$	$0·1173 \times 10^1$	$0·1149 \times 10^1$	$0·1109 \times 10^1$
$\sigma/\sqrt{N}$	Method a	$0·4777 \times 10^{-2}$	$0·3810 \times 10^{-2}$	$0·3373 \times 10^{-2}$	$0·3078 \times 10^{-2}$	$0·2889 \times 10^{-2}$
	Method b	$0·2282 \times 10^{-2}$	$0·1414 \times 10^{-2}$	$0·0958 \times 10^{-2}$	$0·0727 \times 10^{-2}$	$0·05896 \times 10^{-2}$
	Method c	$0·2282 \times 10^{-2}$	$0·1427 \times 10^{-2}$	$0·0979 \times 10^{-2}$	$0·07452 \times 10^{-2}$	$0·06061 \times 10^{-2}$
$1/(\sigma^2 t)$	Method a	$0·0343 \times 10^6$	$0·0397 \times 10^6$	$0·0386 \times 10^6$	$0·0372 \times 10^6$	$0·0352 \times 10^6$
	Method b	$0·1431 \times 10^6$	$0·3921 \times 10^6$	$0·8889 \times 10^6$	$1·591 \times 10^6$	$2·484 \times 10^6$
	Method c	$0·1481 \times 10^6$	$0·4080 \times 10^6$	$0·8893 \times 10^6$	$1·567 \times 10^6$	$2·454 \times 10^6$

obtained by analytic evaluation of the appropriate formulae.[5] In the case that no wear is present, $\lambda(t) \to \lambda_0$, and the unreliability is reduced to 0·062 932.

To compare the sampling methods we have tabulated the computing time, t, the estimated root sample variance and the widely used figure of merit, $1/(\sigma^2 t)$. The quantity $\pm \sigma/\sqrt{N}$, with N being the number of trials, is the estimated 68% confidence interval that appears in all subsequent tables, and the figure of merit is a standard method for comparing the computational efficiencies of alternate Monte Carlo procedures.

As indicated by these and other model problem results, the variant of the self-transition method labeled model (b) is more efficient computationally than variant (c). Both self-transition methods are clearly superior to mode sampling method, model (a). It should be observed that for multiple-component systems mode sampling also becomes more cumbersome, particularly for problems with repair models. Conversely, self-transition becomes more efficient in the presence of preventive maintenance since the resulting reduction in the maximum failure rate over the life of the components reduces the fraction of self-transition. Thus, models (a) and (b) are disregarded for subsequent problems.

In Table 2 are shown single-component unavailability results for both revealed and unrevealed failures. The interval unavailability is tabulated for a 5-yr design life. For the revealed failures a repair rate of $\mu = 10\,\mathrm{yr}^{-1}$ is used, and wear is assumed to accumulate through the repair period as indicated in model c in Fig. 2. For the time-independent failure rate $\lambda = \lambda_0$, the second term is deleted from eqn (10). In the latter case, the results indicate that preventive maintenance has no effect on the

TABLE 2
Single-component Interval Unavailabilities*

	No maintenance	Annual maintenance
Revealed failures‡		
Constant, λ_0	$0·1262 \times 10^{-2} \pm 0·0013 \times 10^{-2}$	$0·1262 \times 10^{-2} \pm 0·0013 \times 10^{-2}$
Increasing, $\lambda(t)$	$0·809 \times 10^{-2} \pm 0·0110 \times 10^{-2}$	$0·1886 \times 10^{-2} \pm 0·0026 \times 10^{-2}$
Unrevealed failures		
Constant, λ_0†	$0·4717 \times 10^{-1} \pm 0·0020 \times 10^{-1}$	$0·0187 \times 10^{-1} \pm 0·0004 \times 10^{-1}$
Increasing, $\lambda(t)$†	$2·1452 \times 10^{-1} \pm 0·0230 \times 10^{-1}$	$0·02525 \times 10^{-1} \pm 0·0007 \times 10^{-1}$

* 5-yr design life; † failure rates from eqn (10); ‡ $\mu = 10\,\mathrm{yr}^{-1}$.

unavailability. When wear is added, by using eqn (10) to represent the time-dependent failure rate, the unavailability increases as would be expected. When annual as-good-as-new preventive maintenance is included on an annual basis, $N = 5$, then the unavailability is reduced significantly, remaining, of course, above the value for which no wear is present.

For unrevealed failures, the unavailability results are smallest for the constant failure rate case, where no wear is present. For the case with wear the annual test/repair is assumed to restore the component to an as-good-as-new condition as in model a of Fig. 3. This modeling is necessary to remain within the Markov framework as discussed in the preceding section. As indicated by the table, the annual test and repair causes a significant decrease in the unavailability whether or not wear is present. For all unrevealed failure calculations here and in what follows it is assumed that the repair time can be ignored compared to the downtime in the unrevealed failed condition.

Two components

We consider next a simple active parallel system consisting of two components in order to illustrate component interactions in the presence of wear, preventive maintenance and of shared-load dependencies. Each component is represented by a failure rate given by eqn (10), either with the last term deleted (the $\lambda = \lambda_0$ time-independent failure rate) or in the wear model with both terms present. The failure and repair parameters are those used for the single-component system. Preventive maintenance, where included, is performed annually on a staggered basis for the duration of the 5-yr design life (i.e. maintenance is performed at 1, 3 and 5 yrs on component one, and at 2 and 4 yrs on component two). The models for revealed and unrevealed failures are the same as above.

The unreliabilities and unavailabilities are given in Tables 3 and 4, respectively. In these calculations the component failures are assumed to be independent for the time-independent failure rate γ_0. For the time-dependent failure rates $\lambda(t)$, both independent and shared loads are given. In the shared load cases the dependency is modeled by assuming an increased rate of wear when only one component is in operation. This is accomplished by replacing θ by $\theta' = \theta - \Delta\theta$ in eqn (10) with $\Delta\theta = 2\cdot5$ yr. In the unreliability calculations repair of the redundant compone .. is allowed until system failure occurs. The increases in unreliability and

TABLE 3
Two-component System Unreliability*

	No maintenance	Maintenance
Revealed failures		
Constant, λ_0	$0.1652 \times 10^{-3} \pm 0.0003 \times 10^{-3}$	$0.1652 \times 10^{-3} \pm 0.0003 \times 10^{-3}$
Increasing, $\lambda(t)$ (independent)	$0.9290 \times 10^{-2} \pm 0.0180 \times 10^{-2}$	$0.0657 \times 10^{-2} \pm 0.0004 \times 10^{-2}$
Increasing, $\lambda(t)$ (shared load)	$0.2320 \times 10^{-1} \pm 0.0044 \times 10^{-1}$	$0.0123 \times 10^{-1} \pm 0.0001 \times 10^{-1}$
Unrevealed failures		
Constant, λ_0	$0.3940 \times 10^{-2} \pm 0.0022 \times 10^{-2}$	$0.0432 \times 10^{-2} \pm 0.0017 \times 10^{-2}$
Increasing, $\lambda(t)$ (independent)	0.1209 ± 0.0012	$0.0736 \times 10^{-2} \pm 0.0046 \times 10^{-2}$
Increasing, $\lambda(t)$ (shared load)	0.2543 ± 0.0020	$0.1262 \times 10^{-2} \pm 0.0081 \times 10^{-2}$

* 5-yr design life.

TABLE 4
Two-component System Interval Unavailability

	No maintenance	Maintenance
Revealed failures		
Constant, λ_0	$0.1640 \times 10^{-5} \pm 0.0017 \times 10^{-5}$	$0.1640 \times 10^{-5} \pm 0.0017 \times 10^{-5}$
Increasing, $\lambda(t)$ (independent)	$0.9177 \times 10^{-4} \pm 0.0290 \times 10^{-4}$	$0.0648 \times 10^{-4} \pm 0.0009 \times 10^{-4}$
Increasing, $\lambda(t)$ (shared load)	$0.2426 \times 10^{-3} \pm 0.0110 \times 10^{-3}$	$0.0123 \times 10^{-3} \pm 0.0002 \times 10^{-3}$
Unrevealed failures		
Constant, λ_0	$0.2010 \times 10^{-2} \pm 0.0021 \times 10^{-2}$	$0.0376 \times 10^{-2} \pm 0.0015 \times 10^{-2}$
Increasing, $\lambda(t)$ (independent)	$0.4085 \times 10^{-1} \pm 0.0071 \times 10^{-1}$	$0.0069 \times 10^{-1} \pm 0.0004 \times 10^{-1}$
Increasing, $\lambda(t)$ (shared load)	$0.7922 \times 10^{-1} \pm 0.0120 \times 10^{-1}$	$0.0109 \times 10^{-1} \pm 0.0007 \times 10^{-1}$

unavailability due to load sharing and to wear, and the corresponding decreases due to maintenance, are apparent from Tables 3 and 4.

Ten-component system

To illustrate the capabilities of the inhomogeneous Markov Monte Carlo formalism we consider next the ten-component system described
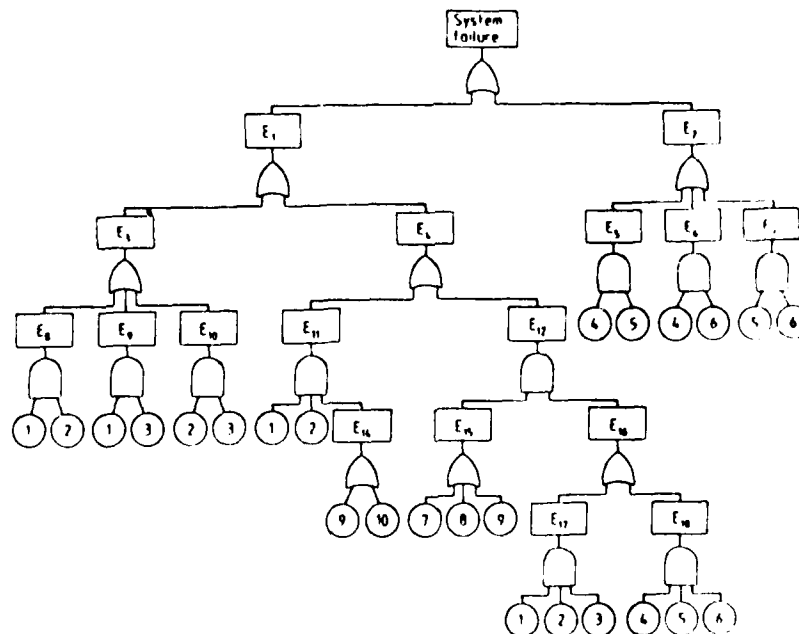
Fig. 5. Fault tree for ten-component problem.

by the fault tree of Fig. 5. The system has been analysed previously[1,2] using the constant failure and repair rate data given in Table 5. The unreliability and unavailability results are given respectively in Tables 6 and 7. In the unreliability calculations, repair is allowed on redundant components until system failure occurs.

Model (1) is a reference calculation with independent revealed failures and using the time-independent failure and repair rate data of Table 5. In this and the succeeding calculations the design life is 1000 hr. Since no wear is present, maintenance has no effect on model (1).

In model (2) wear effects are added to components one through three by representing them with eqn (10). In this λ_0 is determined from Table 5 and the wear is characterized by $\theta_i = 10\,815\,h$ and $m_i = 2.5$. Components one through three constitute a 2/3 subsystem, and we assume load sharing by reducing θ_i of the operating components by $\Delta\theta = 3000\,h$ for each failed component. For the calculations with maintenance, as-good-as-new preventive maintenance is performed at 100-h intervals on a staggered basis (e.g. at 100, 400, ... for component (1) and at 200, 500, ...

TABLE 5
Data for Example Problem

i	Group	$\lambda_i(10^{-5}h^{-1})$	$\mu_i(h^{-1})$
1	1	0·26	0·042
2	1	0·26	0·042
3	1	0·26	0·042
4	2	3·5	0·17
5	2	3·5	0·17
6	2	3·5	0·17
7	3	0·5	0
8	3	0·5	0
9	4	0·8	0
10	4	0·8	0

for component (2) and at 300, 600, . . . for component (3)). Components four through ten are treated is in model (1). The results indicate both the increase in unavailability due to the wear effects and the mitigation of wear by preventive maintenance.

In model (3) components one through three and seven through ten are treated as in model (1). Components four through six are taken to be unrevealed failures by setting $\mu_i = 0$. Components three through six also constitute a 2/3 subsystem. For the calculations with maintenance the same 100-h interval staggered schedule is used as described for components one through three in model (2). Tables 6 and 7 indicate the sharp loss in reliability when unrevealed failures are present, along with the extent of the loss mitigation due to the staggered test and repair schedule.

In model (4) the capability of combining revealed and unrevealed failures as well as wear and preventive maintenance in a single calculation is illustrated. In these simulations components one through three are treated as in model (2), components four through six as in model (3)

TABLE 6
Unreliability for Ten-component System

Model	No maintenance	Maintenance
(1)	$0.4394 \times 10^{-4} \pm 0.0046 \times 10^{-4}$	$0.4394 \times 10^{-4} \pm 0.0046 \times 10^{-4}$
(2)	$0.5367 \times 10^{-4} \pm 0.0078 \times 10^{-4}$	$0.4387 \times 10^{-4} \pm 0.0048 \times 10^{-4}$
(3)	$0.3494 \times 10^{-2} \pm 0.0042 \times 10^{-2}$	$0.0180 \times 10^{-2} \pm 0.0014 \times 10^{-2}$
(4)	$0.4083 \times 10^{-2} \pm 0.0055 \times 10^{-2}$	$0.0373 \times 10^{-2} \pm 0.0015 \times 10^{-2}$

TABLE 7
Unavailability for Ten-component System

Model	No maintenance	Maintenance
(1)	$0.1436 \times 10^{-6} \pm 0.0053 \times 10^{-6}$	$0.1436 \times 10^{-6} \pm 0.0053 \times 10^{-6}$
(2)	$0.2383 \times 10^{-6} \pm 0.0120 \times 10^{-6}$	$0.1522 \times 10^{-6} \pm 0.0061 \times 10^{-6}$
(3)	$0.2339 \times 10^{-2} \pm 0.0036 \times 10^{-2}$	$0.0133 \times 10^{-2} \pm 0.0012 \times 10^{-2}$
(4)	$0.2793 \times 10^{-2} \pm 0.0046 \times 10^{-2}$	$0.0194 \times 10^{-2} \pm 0.0013 \times 10^{-2}$

and the remaining four components as in model (1). Tables 6 and 7 illustrate the synergetic effects of the combined failure and repair models on the system behavior.

6 DISCUSSION

In the foregoing sections the capabilities of inhomogeneous Markov Monte Carlo methods are demonstrated. They allow wear and preventive maintenance to be modeled within the simulation of large systems. Moreover, a limited class of repair models may also be included for both revealed and unrevealed failures. In our illustrations we have not included examples of standby or shared repair crew dependencies, but these also are easily included within the inhomogeneous Markov framework. Likewise, the method is easily extended to account for fixed component downtimes for testing and repair of unrevealed failures as well as for imperfect repair.

A more challenging task is the generalization of the methods to include non-Markov processes. Two of the more important of these are the as-good-as-new repair of revealed failures and the as-good-as-old repair of unrevealed failures, both illustrated in Figs 2 and 3. Inclusion of these phenomena as well as the use of more realistic cumulative damage models for wear are the obvious next step in the development of Monte Carlo simulation of system reliability.

ACKNOWLEDGMENT

E. E. Lewis, Tu Zhuguo

REFERENCES

1. Lewis, E. E. and Boehm, F. Monte Carlo simulation of Markov unreliability models, *Nucl. Engng Des.*, **77** (1984), pp. 49–62.
2. Tu, Z. and Lewis, E. E. Component models in Markov Monte Carlo simulation, *Reliab. Engng*, **13** (1985), pp. 45–61.
3. Larson, H. S. and Shubert, B. O. *Probabilistic Models in Engineering Science*, Vol. II, Wiley, New York, 1979.
4. Gumbel, J. E. *Statistics of Extremes*, Columbia University Press, New York, 1958.
5. Lewis, E. E. *Introduction to Reliability Engineering*, Chapter 8, Wiley, New York (in press).